Resonantlike phenomenon in short-pulse free-electron-laser oscillators with modulated desynchronism

Oscar G. Calderón,* Takuji Kimura, and Todd I. Smith
Stanford Picosecond FEL Center, Hansen Experimental Physics Laboratory, Stanford University, Stanford, California 94305-4085
(Received 8 February 2001; revised manuscript received 24 July 2001; published 18 December 2001)

Recently a new method of controlling the pulse length of a short-pulse free-electron laser (FEL) has been developed. By modulating the synchronism between the optical and electron pulses in the FEL cavity, it was found that the output power and the micropulse length of the FEL beam oscillates at the modulation frequency. In this paper, we study theoretically the behavior of the micropulse length, both in the high loss (steady state) regime and the low loss (limit cycle) regime, when a modulated desynchronism is applied. In order to do this, we analyze the dynamics of a short-pulse FEL oscillator. The modulation frequency value plays an important role in the dynamics. We find that there is a resonantlike phenomenon between the externally applied desynchronism modulation and the limit cycle oscillation without modulation of a free-electron laser.

DOI: 10.1103/PhysRevE.65.016504 PACS number(s): 41.60.Cr, 42.65.Sf, 42.60.Jf

I. INTRODUCTION

Free-electron-laser (FEL) oscillators driven by short electron pulses exhibit a large variety of nonlinear behaviors such as limit cycles, chaos, and super-radiance [1–3]. In FEL oscillators successive electron pulses periodically enter the wiggler, where they copropagate with the stored optical pulse. By varying the desynchronism ($\delta\mathcal{L}$) between the periodic beam injection and the round trip time of the radiation in the cavity, it is possible to control the overlapping between the radiation and electron pulses during many round-trips. In fact, a finite desynchronism $\delta\mathcal{L}$ is necessary to maintain synchronism between electron and optical pulses in the cavity because of the effect of laser lethargy, which causes the centroid of the optical pulse to travel slower than the speed of light in vacuum.

During the past few years there has been much interest in the topic of generating and controlling short FEL pulses [2–8]. Jaroszynski *et al.* [9] and Bakker *et al.* [4] have demonstrated the method of switching an FEL rapidly between high gain and high power regimes using dynamical desynchronism techniques. In a recent work [10,11] it has been shown that modulating the desynchronism provides a new method of controlling the pulse length of an FEL. It was found that the output power and the micropulse length of the FEL beam oscillates at the modulation frequency and that the minimum micropulse length during the cycle can be significantly shorter than that can be obtained without modulation. It was observed using the Stanford midinfrared FEL that when $\delta\mathcal{L}$ is modulated by a few microns at 40 kHz, the measured FEL micropulse length varies as a function of the phase relative to the modulation from a minimum of 300 fs to a maximum of 800 fs. When the modulation is turned off, the micropulse length is about 700 fs. These measurements were made at a wavelength of 5 μm and the relevant FEL parameters are given in Table I. This extreme behavior was related with the operation of the FEL for part of the cycle in the normally inaccessible portion of the output power curve where the gain is less than cavity losses [11,12].

In their work, Jaroszynski *et al.* [13] have shown that the instabilities associated with the limit cycle and chaotic behavior can be examined by periodically adjusting the cavity desynchronism. They observed resonances at limit cycle, the period-multipled frequencies. Limit cycle oscillations in a short-pulse FEL, as a result of a periodic, self-replicating micropulse structure on successive cavity transits, were first predicted by Colson in 1982 [14]. The first conclusive experimental evidence was demonstrated in 1993 [2]. It was shown that the period of the self-sustained oscillation is close to the time a micropulse takes to advance by the slippage distance L_s, i.e., the distance over which the electrons slip back relative to the light field as they traverse the wiggler. Therefore, the value of the characteristic limit-cycle frequency is

$$f_{lc} \simeq \frac{\delta\mathcal{L}}{L_s} f_{FEL}, \tag{1.1}$$

where $1/f_{FEL}$ is the roundtrip cavity time. Piovella *et al.* [7] interpreted this limit cycle as a competition between supermodes. These supermodes are the eigenvectors of the linearized system, that is, in the small gain regime.

TABLE I. Usual Stanford midinfrared FEL parameters.

Electron energy (γ_0)	64.6
Peak electron current (I_b)	13 A
Electron pulse length (L_b)	390 μm
Undulator period (λ_w)	3.1 cm
Number of undulator periods (N_w)	72
Roundtrip cavity time $1/f_{FEL}$	84.6 ns
Normalized wiggler vector potential (a_w)	0.83
Radiation wavelength (λ)	5 μm
Slippage length (L_a)	360 μm
Gain parameter (γ)	0.283
Cavity loss (α_0)	0.03

*Permanent address: Departamento Optica, Universidad Complutense, Cuidad Universitaria s/n, 28040 Madrid, Spain.

In this paper, we study theoretically the behavior of the micropulse length when a small modulation of the desynchronism is applied. We analyze the dynamics of an FEL oscillator both in the high loss and the low loss regime, by means of the short-pulse FEL oscillator equations [7,8,11]. We observe that in the low loss regime, a resonantlike phenomenon between the desynchronism modulation and the intrinsic oscillation without modulation of a free-electron laser occurs. When the frequency of the modulation matches that of the limit-cycle oscillation, a minimum micropulse length is obtained. In the high loss regime, we find that a small modulation modifies the micropulse length dynamics of the FEL. We find a characteristic frequency of modulation that produces a large amplitude of the oscillation of the micropulse length.

II. THEORETICAL MODEL

In a short-pulse FEL oscillator, an optical pulse circulates in a cavity between two reflecting mirrors and interacts with a new electron pulse during each pass along the z axis of a $L_w = N_w \lambda_w$ long wiggler, where λ_w is the wiggler period and N_w the number of wiggler periods. The length of the electron pulse L_b is considered much smaller than the slippage length $L_s = \lambda N_w$, where $\lambda = \lambda_w(1+a_w^2)/(2\gamma_0^2)$ is the resonant wavelength of the radiation, a_w is the normalized wiggler vector potential, and γ_0 the beam energy in rest mass units mc^2. We describe the radiation field in the cavity by its slowly varying complex amplitude $A(z,t)$. The set of FEL equations can be written as (see Refs. [7,8,11])

$$\frac{\partial A}{\partial \tau} - \nu \frac{\partial A}{\partial \xi} + \frac{\alpha}{2} A = \eta B, \tag{2.1}$$

$$\frac{\partial B}{\partial \xi} = -iD, \tag{2.2}$$

$$\frac{\partial D}{\partial \xi} = -A - iSB - 2iQD + 2iQ^2 B, \tag{2.3}$$

$$\frac{\partial Q}{\partial \xi} = -[AB^* + \text{c.c.}], \tag{2.4}$$

$$\frac{\partial S}{\partial \xi} = -2[AD^* + \text{c.c.}], \tag{2.5}$$

where $\eta = 1$ for $0 < \xi < 1$ and $\eta = 0$ elsewhere.

In the above equations, $\xi = (ct - z)/L_s$ is the position within the optical pulse in units of the slippage length. The roundtrip number n is considered as a coarse-grained time variable, $\tau = \gamma n$. $\nu = 2\delta\mathcal{L}/(L_s \gamma)$ is the normalized cavity detuning, $\alpha = \alpha_0/\gamma$, where α_0 is the cavity loss, and $\gamma = (L_b/L_s) g_0$. The small signal gain parameter g_0 is related to the fundamental FEL parameter ρ [15] by $g_0 = (4\pi\rho N_w)^3$. The amplitude A is normalized such that $|A|^2 = 4\pi N_w g_0 P_{in}(z,t)/P_e$, where $P_{in}(z,t)$ is the intracavity optical power, and $P_e = mc^2\gamma_0(I_b/e)$ is the electron beam power, with I_b the peak electron current. We have used in these equations a collective variable description [16], i.e., $B = \langle \exp(-i\theta) \rangle$, $D = \langle p \exp(-i\theta) \rangle$, $Q = \langle p \rangle$, and $S = \langle p^2 \rangle$, where $p = \partial_\xi \theta$, and θ the electron phase.

Equations (2.1)–(2.5) have been solved numerically using a finite difference method. The initial beam is assumed to be monochromatic and unbunched, while the initial energy satisfies the FEL resonant condition. At $\xi = 0$ the initial conditions are given by $B = D = Q = S = 0$. We assume a cavity detuning $\nu > 0$, that means $A(\xi = 1, \tau) = A_0$, where A_0 characterizes the level of spontaneous emission [7,8]. The initial field is assumed to match the level of spontaneous emission, $A(\xi, \tau = 0) = A_0$. We define the dimensionless ouput power P as

$$P(\tau) = \int_{-\infty}^{+\infty} d\xi |A(\xi, \tau)|^2. \tag{2.6}$$

We utilize the experimental parameters of the Stanford midinfrared FEL (see Table I). The value for the gain parameter $\gamma = (L_b/L_s) g_0$ is chosen such that the theoretical growth rate of the optical power in the linear regime matches that of the experimental result. The value is determined to be $\gamma = 0.283$, $g_0 = 0.26$. Further calculation using these values shows good agreement between experiment and theory for the output power vs desynchronism curve (see Ref. [11]). The typical value for the cavity loss is $\alpha_0 = 0.03$, i.e., $\alpha = 0.106$. In the experiment, the cavity loss is determined by fitting the falling edge of the optical macropulse to a logarithmic curve. At this high loss value the FEL operates in the steady state regime (no limit cycle) where a constant intensity and micropulse length is reached. These constant values depend on the cavity detuning ν.

A. Modulated desynchronism

We consider a periodically time-dependent cavity detuning ν as follows:

$$\nu = \nu_0 + \frac{\Delta \nu_m}{2} \sin\left(\frac{2\pi f_m}{\gamma f_{FEL}} \tau \right), \tag{2.7}$$

where ν_0 is the average cavity detuning, and $\Delta\nu_m$ and f_m the modulation amplitude and modulation frequency, respectively. This modulation of the cavity detuning can be achieved using the following property: the magnetic chicanes of the Stanford FEL are nonisochronous, i.e., higher electrons pass through them more quickly than the lower energy ones. The effect is calculated to be 0.03 ps/keV. It means that a modulation of the energy beam is translated into modulation of the electron bunch repetition frequency [10]. If the energy modulation takes the form $(\Delta E/2)\cos(2\pi f_m t)$, where ΔE is the peak-to-peak energy modulation in keV, then the fractional change in the electron bunch repetition frequency is given by

$$\frac{\Delta f}{f_{FEL}} = 1.5 \times 10^{-14} 2\pi f_m \Delta E \sin(2\pi f_m t). \tag{2.8}$$

Since a change in the repetition frequency has the same effect as the cavity length detuning $\delta\mathcal{L}_m = L_{\text{cav}}(\Delta f / f_{FEL})$, with

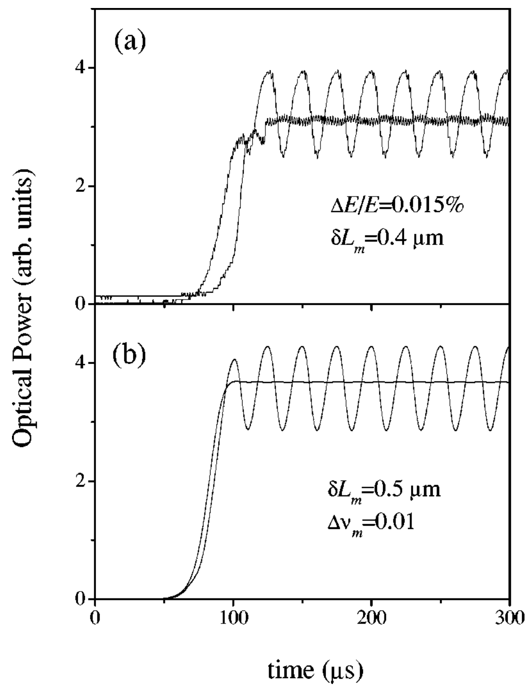

FIG. 1. (a) Experimental and (b) theoretical optical power signals as a function of time with a desynchronism modulation (a) $\delta\mathcal{L}_m = 0.4$ μm, (b) $\delta\mathcal{L}_m = 0.5$ μm. The average desynchronism is $\delta\mathcal{L}_0 = 1.5$ μm ($\nu_0 = 0.03$). The flat curves are for situations without the energy modulation.

$L_{\rm cav} = 12.7$ m, an equivalent cavity length can be calculated from Eq. (2.8). If we compare this cavity length detuning, $\delta\mathcal{L} = \delta\mathcal{L}_0 + \delta\mathcal{L}_m$, with Eq. (2.7) we find that the modulation amplitude is equal to $\Delta\nu_m = 4L_{\rm cav}1.5\times10^{-14}\times 2\pi f_m \Delta E/(\gamma L_s)$.

Figure 1 shows the results obtained in our previous work [11]. Plotted is the temporal evolution of the output power when the electron beam energy is modulated with a small amplitude at a frequency of 40 kHz. Both experimental and theoretical curves are shown. These power signals present a periodic oscillation with the same frequency as the modulation. An intuitive explanation of this behavior is that there is a nearly linear one-to-one mapping of the FEL intensity on the detuning (given by the detuning curve) and therefore it is not too surprising to observe an amplitude modulation of the FEL at the modulation frequency. Since the micropulse length of an FEL operating in steady state is a monotonically increasing function of the cavity detuning parameter, the same argument leads to the expectation that the micropulse length will oscillate periodically at the modulation frequency. When the modulation amplitude becomes large enough, such that the detuning reaches very small values, outside the emission region, where the gain is less than cavity losses, the linear one-to-one mapping disappears and new frequencies appear due to the nonlinearity of the mapping [11]. In this extreme regime it was found that the optimum frequency for obtaining short pulse with high power was around 40 kHz [11,12].

In this paper we look theoretically for a dynamical coupling between the regular operation of the FEL (inside the emission region) and the modulated desynchronism. At the least, such coupling is expected for the small loss case, when the FEL is operating in the limit-cycle regime, as in Ref. [13]. As two periodic movements are taking place, the intrinsic oscillation of the FEL (limit-cyle oscillation) and the external modulation of the cavity detuning, it would be surprising if the system dynamics were not influenced when the two frequencies were nearly the same.

III. SMALL LOSS REGIME, LIMIT-CYCLE REGIME

As we said above, a small loss value leads to a limit-cycle regime, where the output power develops a periodic oscillation. In Ref. [7] is shown a phase diagram (see Fig. 7 of Ref. [7]) in α and ν of the different regimes, where the region leading to limit cycles can be seen. We consider the typical Stanford midinfrared FEL parameters (see Table I) but in order to get the limit-cycle regime we decrease the cavity loss to $\alpha_0 = 0.0113$. Figure 2 shows the optical power, averaged over one limit-cycle period (dotted line), together with the minimum and maximum of the oscillation (dashed line) as a function of the cavity detuning ν. The limit-cycle regime occurs over the whole range of the detuning curve. The frequency of this limit-cycle oscillation f_{lc} is a function of detuning ν, as shown in Fig. 2 with a solid line. This frequency follows a linear behavior with the cavity detuning in agreement with Eq. (1.1), i.e., $f_{lc} \simeq 0.5\gamma f_{FEL}\nu$.

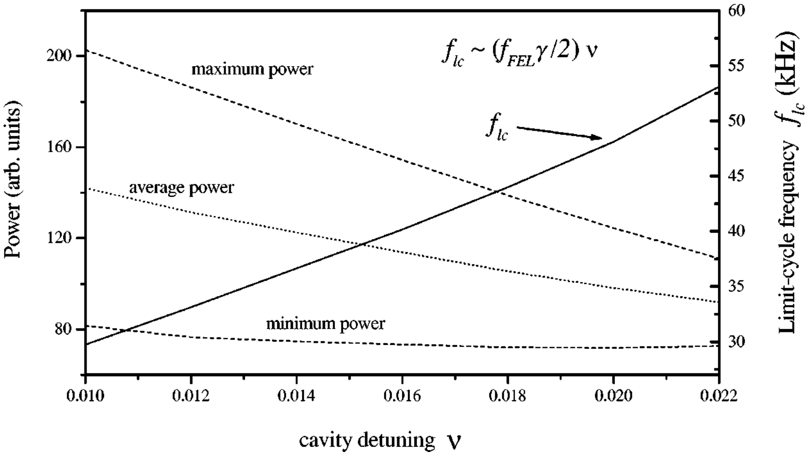

FIG. 2. Optical power P, averaged over a limit-cycle period (dotted line), and maximum and minimum amplitudes (dashed lines), vs cavity detuning for $\alpha_0 = 0.0113$. The limit cycle oscillation frequency (solid line) vs cavity detuning is also shown.

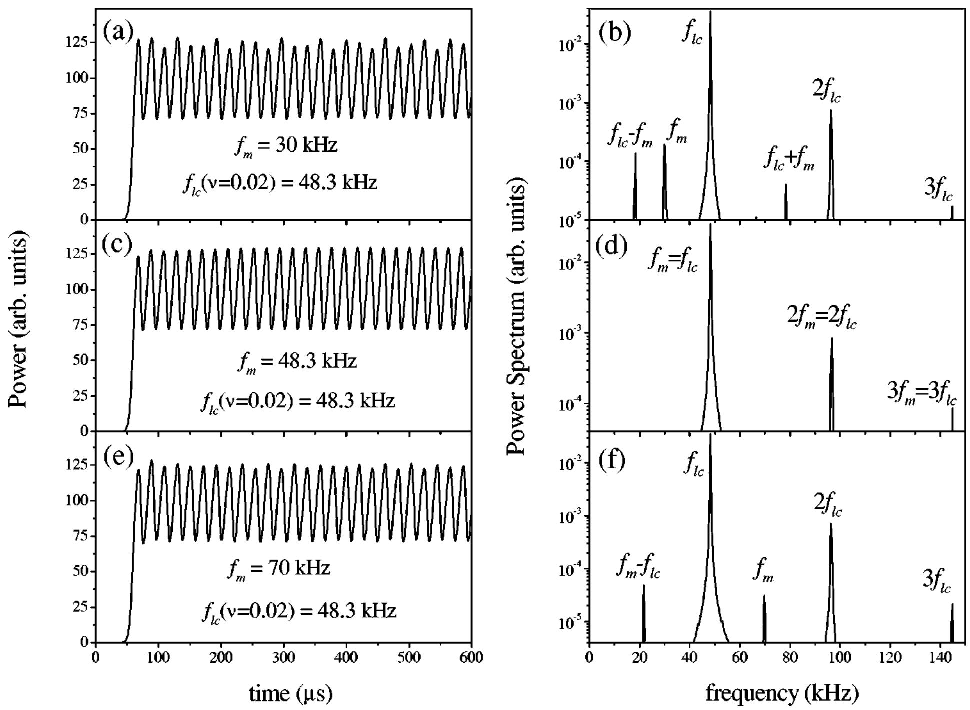

FIG. 3. Temporal evolution of the optical power (a), (c), (e) and its power spectrum (b), (d), (f) for a modulation frequency (a), (b): $f_m=30$ kHz; (c), (d); $f_m=48.3$ kHz; (e), (f): $f_m=70$ kHz. The parameters are $\alpha_0=0.0113$, $\nu_0=0.02$, and $\Delta\nu_m=0.002$.

A. Modulated desynchronism in small loss regime

In this section we study theoretically the effect of modulated desynchronism in the small loss regime. We study the behavior of the optical power when a small modulation amplitude is applied ($\Delta\nu_m=0.002$ or $\delta\mathcal{L}_m=0.1$ μm) at different modulation frequencies ($f_m=10$–80 kHz). Figure 3 shows the temporal evolution of the optical power and its corresponding power spectrum for different modulation frequencies at $\nu_0=0.02$. For $f_m=30$ kHz and $f_m=70$ kHz, a multiperiodic oscillation occurs in the optical power [see Figs. 3(a), 3(b), 3(e), and 3(f)], being the two fundamental frequencies the frequency of the modulation [Figs. 3(a) and 3(b) $f_m=30$ kHz and Figs. 3(e) and 3(f), $f_m=70$ kHz] and the corresponding limit-cycle frequency of the FEL without modulation at $\nu=0.02$, $f_{lc}\simeq 48.3$ kHz (see Fig. 2). Combinations of both frequencies, $f_m\pm f_{lc}$, are also found. This result means that the well-known periodic oscillation of FEL without modulation (limit-cycle oscillation) still appears when the cavity detuning is modulated. Therefore, two different movements are governing the FEL dynamics; the intrinsic limit-cycle oscillation, that is the natural oscillation of the FEL, and a periodic movement due to the external forcing of the cavity detuning (modulated desynchronism). Now, a natural question arises; What will happen when the modulation frequency matches the frequency of the limit-cycle oscillation? The result is shown in Figs. 3(c) and 3(d). Now, only one peak and its harmonics appear. In this case the frequency of the external forcing of the cavity detuning matches the intrinsic oscillation of the FEL, therefore a resonance phenomenon is expected. These resonances were found experimentally by Jaroszynski *et al.* [13] where they measured the peak macropulse power as a function of the modulation period of the desynchronism.

We are going to study how this resonance affects the behavior of the micropulse. It is well known that the micropulse has structure in the limit-cycle regime without cavity detuning modulation. As was remarked by Chaix in Ref. [8], for each period the micropulse evolves in the following way: the gain drops first at the tail, while the optical field still grows at the head, reaches a high level, and is finally evacuated forward by the cavity detuning, thus allowing a new start-up of the gain at the tail. In order to analyze the behavior of the micropulse we compute the micropulse length at each time taking the maximum peak and calculating the length of this subpulse. Figure 4 (solid line) shows the micropulse structure when the minimum micropulse length, $L_{\min}\simeq 452$ fs, is reached at $\nu=0.02$.

We analyze the minimum micropulse length, $L_{\min}$ when the cavity detuning is periodically modulated. $L_{\min}$ versus the modulation frequency is shown in Fig. 5 for a small modulation amplitude $\Delta\nu_m=0.002$ ($\delta\mathcal{L}_m=0.1$ μm), at two different average cavity detunings. In both cases, $L_{\min}$ is nearly constant and close to the value without modulation through

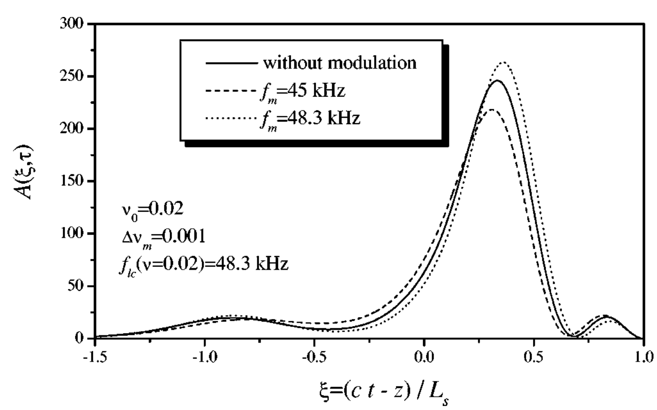

FIG. 4. Micropulse shape, $A(\xi,\tau)$, corresponding to the minimum micropulse length without modulation (solid line), and with modulation the minimum micropulse length (dotted line) at $f_m=48.3$ kHz, and the maximum (dashed line) at $f_m=45$ kHz.

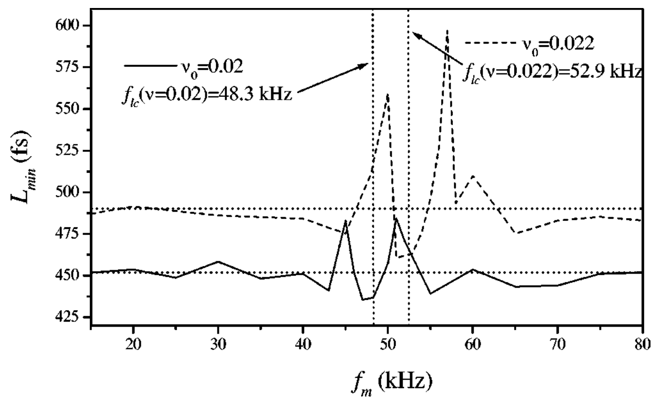

FIG. 5. Minimum micropulse length $L_{\rm min}$ vs modulation frequency, f_m, for low loss $\alpha_0=0.0113$ at two different average cavity detunings; $\nu_0=0.02$ (solid line) and $\nu_0=0.022$ (dashed line). The modulation amplitude is $\Delta\nu_m=0.002$. The dotted lines represent the minimum micropulse length without modulation.

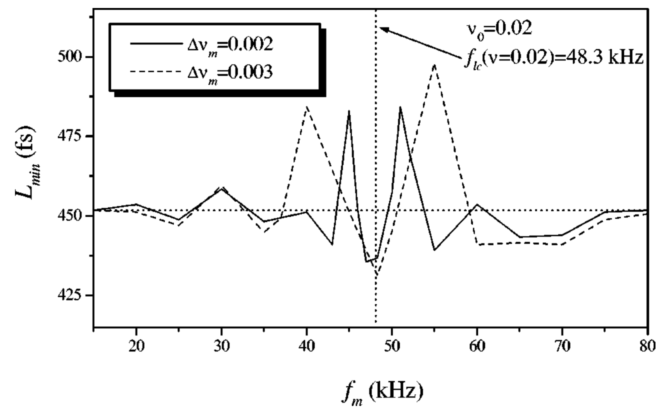

FIG. 6. Minimum micropulse length $L_{\rm min}$ vs modulation frequency, f_m, for low loss $\alpha_0=0.0113$ at two different modulation amplitude; $\Delta\nu_m=0.002$ (solid line) and $\Delta\nu_m=0.003$ (dashed line). The average cavity detuning is $\nu_0=0.02$. The dotted line represent the minimum micropulse length without modulation.

most of the range of modulation frequencies. Hovever, when f_m comes close to the limit-cycle frequency associated with its average cavity detuning f_{lc}, $L_{\rm min}$ changes noticeably. A minimum value is reached when f_m matches the limit-cycle frequency. Its corresponding micropulse shape can be seen in Fig. 4 (dotted line). Around this minimum [see Fig. 5] two maximum peaks, situated on both sides of the minimum, appear. The micropulse shape giving one of this maximum peaks is shown in Fig. 4 (dashed line).

Before continuing, we would like to point out that this resonant curve (f_m,$L_{\rm min}$) is similar to the typical resonant curve that appears in the Farady effect [17] when plotting the rotated angle of the plane of polarization vs field frequency. This curve is the resultant of two equal and opposite optical rotatory dispersion curves for two adjacent electronic absortion bands, since the natural oscillation frequency of the electron splits into two lines symmetrically disposed about the central line. Therefore, it shows a minimum at the original frequency and two maximum peaks situated at the two splitting lines. The same behavior is found in our problem. As we mention above, the limit-cycle frequency is a function of the cavity detuning, $f_{lc}\propto\nu$, (see Fig. 2). Therefore, when we modulate the cavity detuning, the natural frequency of the FEL without modulation f_{lc} gets broader around its value so there is a range of modulation frequencies f_m that resonate. The distance between the two maximum peaks roughly measures the spectral width of the limit-cycle oscillation. Then, this spectral width should depend on the modulation amplitude value since more frequencies are incorporated to the limit-cycle oscillation as $\Delta\nu_m$ increases. As we can see in Fig. 5, where the modulation amplitude is the same for both curves, the distance between the two maximum peaks is almost the same for both curves, in agreement with our hypothesis. To check if this distance depends on the modulation amplitude we plot in Fig. 6 L_{min} vs f_m at $\nu_0=0.02$ for two different modulation amplitudes. It can be seen in this figure how the spectral width, i.e., the distance between the two maximum peaks, increases as the modulation amplitude increases.

From the previous results it is clear that a type of resonant phenomenon is taking place in the small loss regime, since two periodic oscillations coexist. However, in the high loss regime both the power and the micropulse shape show a steady state. Therefore, it is not obvious that any kind of resonance phenomenon will occur when an external modulation of the cavity detuning is applied.

IV. HIGH LOSS REGIME, STEADY STATE REGIME

In this section we theoretically study the FEL dynamics in the high loss regime. We consider a high loss value $\alpha_0=0.03$. The Stanford FEL typically operates with this loss value. In this regime, both the output power and the micropulse shape remains constant, i.e., the FEL operates in steady state, and shows no limit-cycle behavior. In Fig. 7 we plot the optical power and the micropulse length as a function of the cavity detuning. We can see in this figure how the micropulse length follows a roughly linear behavior with the cavity detuning, $L\propto\nu$.

A. Modulated desynchronism in high loss regime

In this section we study theoretically the effect of the modulated desynchronism in the high loss regime. We ana-

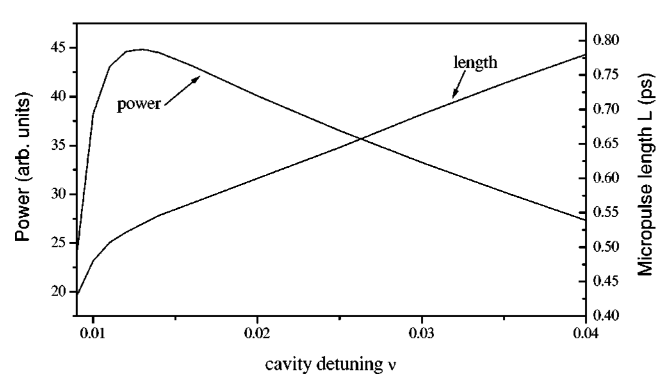

FIG. 7. Optical power P and micropulse length vs cavity detuning for high loss $\alpha_0=0.03$.

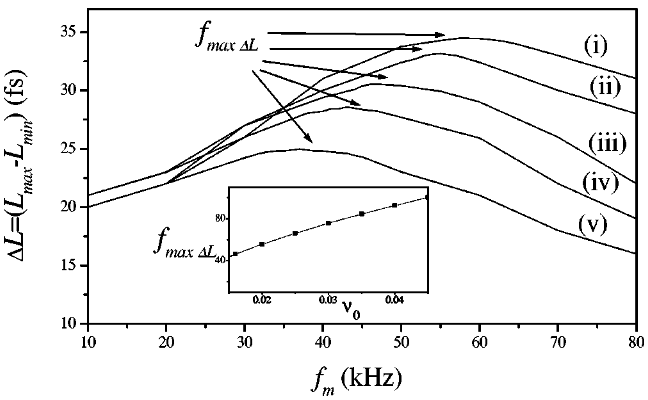

FIG. 8. Amplitude of the micropulse length oscillation, $\Delta L \equiv L_{\max} - L_{\min}$, vs modulation frequency f_m for high loss $\alpha_0 = 0.03$ at different average cavity detuning; (i) $\nu_0 = 0.03$, (ii) $\nu_0 = 0.028$, (iii) $\nu_0 = 0.025$, (iv) $\nu_0 = 0.023$, and (v) $\nu_0 = 0.02$. The modulation amplitude is $\Delta \nu_m = 0.002$. The inset shows $f_{\max\,\Delta L}$ vs the average cavity detuning ν_0.

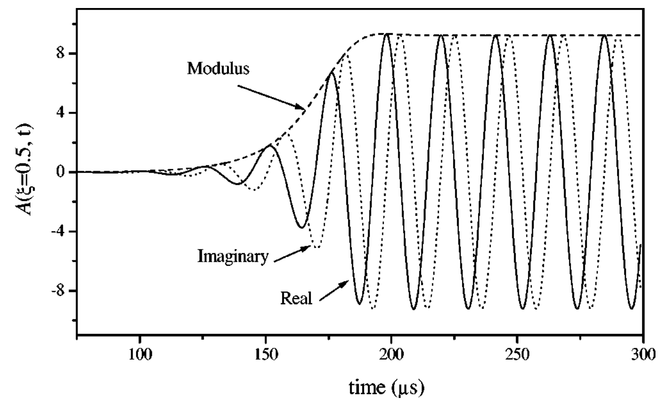

FIG. 9. The temporal evolution of the real part (solid line), the imaginary part (dotted line), and the modulus (dashed line) of the complex optical field, $A(\xi,t)$, at an arbitrary position ($\xi = 0.5$) for a cavity detuning $\nu = 0.016$ ($\delta \mathcal{L} = 0.8$ μm).

lyze the micropulse length when the modulation is applied. As we saw in Fig. 7, the micropulse length changes with the cavity detuning. For very small modulation frequency the system follows the detuning adiabatically, and the micropulse length reaches a minimum at the minimum detuning and vice versa. We assume a very small amplitude of modulation, $\Delta \nu_m = 0.002$ ($\delta \mathcal{L}_m = 0.1$ μm). For the adiabatic case (f_m very small) this detuning modulation gives an oscillation of the micropulse length with an amplitude approximately equal to $\Delta L \equiv (L_{\max} - L_{\min}) \simeq 20$ fs (see Fig. 7). In Fig. 8 we plot the amplitude of the micropulse length oscillation vs the modulation frequency. Each curve corresponds to a different average cavity detuning. Small modulation frequencies present the expected abiabatic behavior, with an amplitude $\Delta L \simeq 20$ fs. An enhancement of the oscillation amplitude in the micropulse length occurs for modulation frequencies between 30 and 65 kHz. In this frequency range the oscillation amplitude of the micropulse length increases dramatically above the adiabatic case. We define the modulation frequency that gives the maximum ΔL as $f_{\max\,\Delta L}$. As we can see in Fig. 8 this frequency value follows a linear behavior with the average cavity detuning, as does the limit-cycle oscillation in the low loss case. This result suggests that some sort of resonance phenomenon is taking place between the internal dynamics of the FEL and the driven desynchronism modulation. Then a natural question arises: what is the interaction? Since in this high loss regime a steady state is present, the internal frequency should be present on the temporal evolution of the complex optical field. Therefore, the solutions of Eqs. (2.1)–(2.5) with constant amplitude (for the steady state) are of the form [8]

$$A(\xi,\tau) = \exp(i 2\pi\Omega t) A(\xi) \equiv \exp\left(i \frac{2\pi\Omega}{f_{FEL}\gamma} \tau \right) A(\xi), \tag{4.1}$$

where Ω is the frequency of the complex optical field oscillation. We plot in Fig. 9 the temporal evolution of the real part, the imaginary part, and the modulus of the complex optical field at an arbitrary position ($\xi = 0.5$) for a cavity detuning $\nu = 0.016$ ($\delta \mathcal{L} = 0.8$ μm). Here we see that both Re $A(\xi = 0.5, t)$ and Im $A(\xi = 0.5, t)$ exhibit an oscillatory evolution with a period close to 22 μs. Since the phase of the real and imaginary components are separated by a constant $\pi/2$, the modulus describes a steady state behavior in agreement with Eq. (4.1). It means a slow natural frequency of the complex optical field is present even in the steady state regime. As this oscillation does not appear in the modulus of the optical field, $|A(\xi,\tau)|$, it does not affect the micropulse shape and the output power evolution. To check if this natural frequency Ω is responsible for the resonance phenomenon found in the high loss regime, we have analyzed its behavior with cavity detuning and found the same linear dependence as for $f_{\max\,\Delta L}$. These results are consistent with the interpretation that the micropulse length oscillation is most strongly influenced by the modulation when its frequency is near the natural oscillation frequency of the complex optical field $A(\xi,\tau)$ that appears without modulation.

We point out that the natural oscillation frequency of the complex optical field Ω exhibits the same behavior as the limit-cycle frequency that appears in the low loss regime, suggesting a strong connection between the two oscillations.

V. CONCLUSIONS

We have studied theoretically the effect of modulated desynchronism and its relation to intrinsic FEL dynamics. The modulation of desynchronism provides a new method of controlling the pulse length of an FEL [10,11]. The minimum micropulse length during a cycle can be significantly shorter than can be obtained without modulation. We have studied the behavior of the micropulse length when a small modulation of the desynchronism is applied. We analyzed the dynamics of an FEL oscillator both in the high loss (steady state) and the low loss (limit-cycle) regimes by means of the short-pulse FEL oscillator equations. In the low loss regime, a resonantlike phenomenon between the external modulation of desynchronism and the intrinsic limit-cycle oscillation of the FEL takes place. When the modulation frequency matches the limit-cycle frequency a minimum of the micropulse length is found. This minimum micropulse length is reached in each cycle of the modulation signal.

In the high loss case, when desynchronism modulation is applied, we find that the oscillation of the micropulse length is maximum for modulation frequencies in the tens of kHz, and that the modulation frequency that provides the maximum effect is a linear function of the detuning length. In this high loss case, where the FEL shows a steady state without modulation, the natural periodic movement that couples with the external cavity detuning modulation is the oscillation of the complex optical field. This frequency does not affect the output power and the micropulse shape. We found that this natural frequency shows the same behavior as the limit-cycle frequency, suggesting a connection between the two.

ACKNOWLEDGMENTS

This work was supported in part by ONR Grant No. N000140-94-1-1024. O.G.C. is supported by Becas Complutense ‘‘del Amo’’ and Becas Complutense ‘‘Flores Valles.’’ O.G.C. wishes to thank Dr. F. Carreño, Dr. M. Antón, and Dr. I. Gonzalo for fruitful discussions about Faraday effect.

[1] S. J. Hahn and J. K. Lee, Phys. Rev. E **48**, 2162 (1993).
[2] D. A. Jaroszynski, R. J. Bakker, A. F. G. van der Meer, D. Oepts, and P. W. van Amersfoort, Phys. Rev. Lett. **70**, 3412 (1993).
[3] D. A. Jaroszynski, P. Chaix, N. Piovella, D. Oepts, G. M. H. Knippels, A. F. G. van der Meer, and H. H. Weits, Phys. Rev. Lett. **78**, 1699 (1997).
[4] R. J. Bakker, G. M. H. Knippels, A. F. G. van der Meer, D. Oepts, D. A. Jaroszynski, and P. W. van Amersfoort, Phys. Rev. E **48**, R3256 (1993).
[5] N. Piovella, Phys. Rev. E **51**, 5147 (1995).
[6] G. Shvets and J. S. Wurtele, Phys. Rev. E **56**, 3606 (1997).
[7] N. Piovella, P. Chaix, G. Shvets, and D. A. Jaroszynski, Phys. Rev. E **52**, 5470 (1995).
[8] P. Chaix, N. Piovella, and G. Gregoire, Phys. Rev. E **59**, 1136 (1999).
[9] D. A. Jaroszynski, A. F. G. van der Meer, P. W. van Amersfoort, and W. B. Colson, Nucl. Instrum. Methods Phys. Res. A **296**, 480 (1990).
[10] T. I. Smith and T. Kimura, in *Proceedings of the 20th International Free Electron Lasers Conference, Williamsburg, Virginia, 1998*, edited by G. R. Neil and S. V. Benson (Elsevier, Amsterdam, 1999).
[11] O. G. Calderón, T. Kimura, and T. I. Smith, Phys. Rev. ST Accel. Beams **3**, 90 701 (2000).
[12] O. G. Calderón, T. Kimura, and T. I. Smith, Nucl. Instrum. Methods Phys. Res. A (to be published).
[13] D. A. Jaroszynski, D. Oepts, A. F. G. van der Meer, and P. Chaix, Nucl. Instrum. Methods Phys. Res. A **407**, 407 (1998).
[14] W. B. Colson, Phys. Quantum Electron. **8**, 457 (1982).
[15] R. Bonifacio, C. Pellegrini, and L. Narducci, Opt. Commun. **50**, 373 (1984).
[16] R. Bonifacio, F. Casagrande, and L. De Salvo Souza, Phys. Rev. A **33**, 2836 (1986).
[17] The Farady effect is a magneto-optical phenomenon that consists of the rotation of the plane of polarization of a plane-polarized light beam when it passes through a medium in a magnetic field. The magnetic optical rotation can be described in terms of different refractive indices for left and right circularly polarized light. L. D. Barron, *Molecular Light Scattering and Optical Activity* (Cambridge University Press, New York, 1982); L. Levi, *Applied Optics* (Wiley, New York, 1980).